Axial Dispersion for Turbulent Flow with a Large Radial Heat Flux

In 1954, G. I. Taylor derived an expression for an apparent axial dispersion coefficient for mass transfer in turbulent flow under the assumption of no mass transport between the tube wall and the fluid inside the tube. Taylor's expression has been verified in a number of experimental efforts for both mass and heat transfer.

This work extends the results of Taylor to the case where there is a significant transport of heat from the tube wall to the fluid in the tube, such as in a double-pipe heat exchanger. Using an analysis similar to Taylor's, it is shown that the apparent axial dispersion coefficient is about one order of magnitude larger when a large radial gradient is present. Experimental data on a double-pipe heat exchanger show significantly improved correspondence with a dispersion-based model of the system when the larger dispersion coefficients are used.

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Introduction

The concept of an apparent axial dispersion is useful in several areas of transport phenomena. In particular, the employment of this concept in cylindrical geometries allows the use of a mathematical model that involves only the longitudinal space variable even when radial effects cannot be neglected. Therefore, a significant simplification of the model is possible.

G. I. Taylor (1954) derived an expression for an apparent axial dispersion coefficient for mass transfer in turbulent flow under the assumption that radial gradients were caused only by the existence of nonflat velocity profiles. Aris (1956) showed that Taylor's result applied under less restrictive conditions than originally reported. Goldschmidt and Householder (1969) pointed out that the experimental work of Sittel et al. (1968) offered further confirmation of Taylor's expression for the apparent axial dispersion coefficient.

Rodehorst and Law (1970) confirmed that Taylor's model is equally valid for predicting apparent axial dispersion coefficients for heat transfer. This work involved introducing a temperature pulse into an insulated pipe and observing the decay of the pulse at several positions down the pipe from the inlet. Conditions of these experiments were analogous to those of Taylor and others for mass transfer.

Other pertinent work on the axial dispersion model, primarily in the area of solutions to the boundary value problems for laminar flow, has been reported by Gill (1967), Gill and Sankarasubramanian (1970, 1971), and Vrentas and Vrentas (1988), to mention only a few.

Beckman (1969), in attempting to model the unsteady state behavior of a double-pipe heat exchanger in response to a temperature pulse at the tube side inlet, found that very large axial dispersion coefficients were required. That is, dispersion coefficients, determined by nonlinear regression analysis, were an order of magnitude larger than those predicted by the Taylor equation in order that the sum of squares of residuals between the exchanger model and experimental data was minimized. The mathematical model was based on the earlier work of Rodehorst and Law (1970), which had confirmed Taylor's model in an analogous heat transfer system. It was suspected that this discrepancy was caused by the large radial heat flux that exists in the case of the double pipe exchanger. In the heat exchanger, the large radial heat flux causes a large radial temperature gradient and induces a significant axial temperature gradient. An analogous mass transfer system can be envisioned as one in which the tube is soluble in the flowing fluid and is composed of the same material as that injected at the tube inlet. These observations led to the careful consideration of the

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assumptions inherent in Taylor's derivation of the apparent axial dispersion coefficient and how these might be altered to conform to systems with a large radial flux.

In this paper, an expression is derived for the apparent axial dispersion coefficient in the presence of a large radial heat flux. For typical flow conditions, the new result indicates that axial dispersion coefficients in the presence of a large radial heat flux should be about an order of magnitude larger than those when the radial flux is negligible. While the double-pipe heat exchanger is not the ideal experimental system to prove the validity of the resulting equations, the utility of the new result is demonstrated by the vastly improved agreement between a one-dimensional model and experimental data for the exchanger.

Derivation of the Apparent Axial Dispersion Coefficient

It is desired to predict from basic physical parameters (for example, the Reynolds number) proper values of the apparent axial dispersion coefficient for a turbulent flow system subjected to large radial heat flux (and consequently having large radial and axial temperature gradients). An example of such a system is the double-pipe heat exchanger.

The development below follows Taylor's original work. The Reynolds analogy is assumed, that is, that the transport of mass, energy, and momentum by turbulence are analogous. Consequently, their turbulent transport coefficients are equal. For a circular pipe this transport coefficient is given by Taylor (1954) as

$$\epsilon = azv_*/f'(z) \tag{1}$$

where $f(z)$ is the logarithmic function from von Karman's generalized velocity profiles and $f'(z)$ is the derivative of $f(z)$ with respect to z.

Neglecting the molecular and eddy diffusivity of energy in the longitudinal direction, the equation for conservation of energy for such a system can be shown to be

$$\frac{\partial}{\partial r}\left[(\epsilon + \alpha) r \frac{\partial T}{\partial r}\right] = r\left(u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial t}\right) \tag{2}$$

Adding and subtracting the term $rv(\partial T/\partial x)$, Eq. 2 can be rewritten as follows:

$$\frac{\partial}{\partial r}\left[(\epsilon + \alpha) r \frac{\partial T}{\partial r}\right] = r(u - v)\frac{\partial T}{\partial x} + r\left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial x}\right) \tag{3}$$

Assuming that $\partial T/\partial x$ and $\partial T/\partial t$ are independent of radius, Eq. 3 may be integrated over the radius from $r = 0$ to $r = a$. At the tube wall, where $r = a$, $\epsilon = 0$ and $aq/\rho c_p = (\alpha r \partial T/\partial r)^{r=a}$. At the center of the tube $\partial T/\partial r = 0$. Noting that the integral of the first term on the righthand side of Eq. 3 is zero, the result of the integration may be expressed as follows:

$$\frac{aq}{\rho c_p} = \frac{a^2 v}{2}\frac{\partial T}{\partial x} + \frac{a^2}{2}\frac{\partial T}{\partial t} \tag{4}$$

From Eq. 4,

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = \frac{2q}{a\rho c_p} \tag{5}$$

Taylor, in his development, considers the case where the rate of change of T at a point that moves with velocity v is zero. That is,

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x} = 0$$

Since the molecular diffusivity is very small compared to the eddy diffusivity (except near the tube wall) and replacing r by az, Eqs. 1 and 5 may be substituted into Eq. 3 to yield the following:

$$\frac{\partial}{\partial z}\left[\frac{z^2}{f'(z)}\frac{\partial T}{\partial z}\right] = z\frac{(u - v)}{v_*}a\frac{\partial T}{\partial x} + \frac{2qz}{\rho c_p v_*} \tag{6}$$

Following Taylor's analysis, it is assumed that $\partial T/\partial x$ is independent of z. Since $\partial T/\partial z = 0$ at $z = 0$ (centerline of the pipe), integration of Eq. 6 once with respect to z gives

$$\frac{z^2}{f'(z)}\frac{\partial T}{\partial z} = a\frac{\partial T}{\partial x}\int_0^z z\frac{(u - v)}{v_*}dz + \frac{2q}{\rho c_p v_*}\int_0^z z\,dz \tag{7}$$

Upon substituting for $(u - v)/v_*$ from von Karman's generalized velocity profiles and completing the integration of the last term, Eq. 7 becomes

$$\frac{z^2}{f'(z)}\frac{\partial T}{\partial z} = a\frac{\partial T}{\partial x}\int_0^z z[f(z) - 4.25]\,dz + \frac{qz^2}{\rho c_p v_*} \tag{8}$$

Rearrangement and further integration of Eq. 8 yields

$$T = T_0 - a\frac{\partial T}{\partial x}\int_0^z \frac{f'(z)}{z^2}\int_0^z z[f(z) - 4.25]\,dz\,dz + \frac{q}{\rho c_p v_*} \tag{9}$$

The heat flux across a section by dispersion can be calculated as

$$Q = \rho c_p \int_0^a 2\pi r(u - v)T\,dr$$
$$= 2\pi a^2 \rho c_p \int_0^1 z(u - v)T\,dz \tag{10}$$

Substituting into Eq. 10 for $(u - v)$ from von Karman's generalized velocity profiles yields, after rearranging,

$$\frac{Q}{\rho c_p} = 2\pi a^2 v_* \int_0^1 z[f(z) - 4.25]T\,dz \tag{11}$$

Since T_0 (temperature at $z = 0$) is independent of z,

$$\int_0^1 z(u - v)T_0\,dz = 0$$

Therefore, substituting for T from Eq. 9 into Eq. 11 leads to

$$\frac{Q}{\rho c_p} = \left\{2\pi a^3 v_* \frac{\partial T}{\partial x} \int_0^1 z[f(z) - 4.25] \cdot \int_0^z \frac{f'(z)}{z^2} \int_0^z z[f(z) - 4.25]\, dz\, dz\, dz\right\} - \left\{2\pi a^2 \frac{q}{\rho c_p} \int_0^1 z[f(z) - 4.25]\right\} \tag{12}$$

The first bracketed term of the above expression was evaluated by Taylor (1954). The second term was computed by numerically integrating (via trapezoidal approximation) the data given in the same reference. The resulting expression is

$$\frac{Q}{\rho c_p} = -10.06\pi a^3 v_* \frac{\partial T}{\partial x} - 10.06\pi a^2 \frac{q}{\rho c_p} \tag{13}$$

Since by definition,

$$\frac{Q}{\rho c_p} = -\frac{D_L}{\rho c_p} \pi a^2 \frac{\partial T}{\partial x} \tag{14}$$

then the apparent dispersion diffusivity is given by

$$\frac{D_L}{\rho c_p} = 10.06 a v_* + 10.06 \frac{q}{\rho c_p (\partial T/\partial x)} \tag{15}$$

The first term on the righthand side of Eq. 15 is the same as that derived by Taylor and is the apparent axial dispersion coefficient for the case of no heat transfer at the wall. The second term on the righthand side is the contribution of the radial gradient produced by the heat flux at the wall. Assuming a low energy transient disturbance (e.g., a temperature pulse of short duration), then to an acceptable approximation the following steady state energy balance applies:

$$\frac{q}{\rho c_p (\partial T/\partial x)} = \frac{av}{2} \tag{16}$$

Substituting Eq. 16 into Eq. 15 and adding Taylor's (1954) correction of 0.05 av_* to account for axial dispersion due to axial diffusion gives

$$\frac{D_L}{\rho c_p} = 10.1 a v_* + 5.03 a v \tag{17}$$

Equation 17 gives the primary result of this paper. It is derived for a uniform heat flux at the tube wall and for the case of a longitudinal temperature gradient that is primarily from a steady state heat transfer from the tube wall. However, the radial temperature profile in the turbulent flow region of a pipe should not be greatly affected by moderate variations of the heat flux. Consequently, Eq. 17 should provide a good approximation of the dispersion coefficient for cases in which the heat flux varies moderately with longitudinal distance. Note that Eq. 17 allows the computation of the apparent axial dispersion coefficient in the presence of a large radial heat flux from only the tube radius and the Reynolds number. In a later section Eq. 17 is used Vo predict the values of D_L in a one-dimensional model ofa double-pipe heat exchanger, where the hydraulic radius is used on the shell side. The excellent agreement between the model and experimental data confirms the utility of Eq. 17.

It is interesting to compare the magnitude of the D_L and D_T, the adiabatic turbulent dispersion coefficient of Taylor, which is given by

$$\frac{D_L}{\rho c_p} = 10.1 a v_* \tag{18}$$

To an acceptable approximation, Eq. 17 may be written as

$$\frac{D_L}{\rho c_p} = 10.1 a v_* + 5.05 a v \tag{19}$$

Thus, a suitable estimate of the D_L/D_T ratio is given by

$$\frac{D_L}{D_T} = 1 + \frac{v}{2v_*} = 1 + \frac{1}{\sqrt{2F}} \tag{20}$$

Since typical friction factors for highly developed turbulence in rough pipes are in the range 0.005 to 0.01, then approximately it is expected that

$$8 \le \frac{D_L}{D_T} \le 11$$

This result shows that the apparent axial dispersion coefficient in the presence of a large radial heat flux should be about an order of magnitude larger than that for an adiabatic system.

Mathematical Model of the Exchanger

A mathematical description of the countercurrent, liquid-liquid, double-pipe heat exchanger can be obtained from energy balances over the liquid phases and the tube walls. Details of the model development were given by Beckman and Law (1971) as well as in the original work of Beckman (1969). The model was derived under the following assumptions:

1. A one-dimensional model adequately describes the heat transfer in the exchanger.
2. The exchanger contains two incompressible liquid phases in single-pass, countercurrent flow.
3. The physical properties of the fluids and pipe walls do not vary appreciably with temperature.
4. Heat transfer coefficient and thermal dispersion coefficient variations with temperature are negligible.
5. The outer tube is perfectly insulated from its surroundings.

The resulting partial differential equations are summarized below:

Tube fluid energy balance

$$\frac{D_1}{\rho_1 c_{p_1}} \frac{\partial^2 U}{\partial x^2} - v_1 \frac{\partial U}{\partial x} - \frac{h_1 P_1}{A_1 \rho_1 c_{p_1}} (U - W) = \frac{\partial U}{\partial t} \tag{21}$$

Inner tube wall energy balance

$$\frac{k_2}{\rho_2 c_{p_2}} \frac{\partial^2 W}{\partial x^2} + \frac{h_1 P_1}{A_2 \rho_2 c_{p_2}} (U - W) - \frac{h_2 P_2}{A_2 \rho_2 c_{p_2}} (W - V) = \frac{\partial w}{\partial t} \quad (22)$$

Shell fluid energy balance

$$\frac{D_3}{\rho_3 c_{p_3}} \frac{\partial^2 V}{\partial x^2} + v_3 \frac{\partial V}{\partial x} + \frac{h_2 P_2}{A_3 \rho_3 c_{p_3}} (W - V) - \frac{h_3 P_3}{A_3 \rho_3 c_{p_3}} (V - Z) = \frac{\partial V}{\partial t} \quad (23)$$

Outer wall energy balance

$$\frac{k_4}{\rho_4 c_{p_4}} \frac{\partial^2 Z}{\partial x^2} + \frac{h_3 P_3}{A_4 \rho_4 c_{p_4}} (V - Z) = \frac{\partial Z}{\partial t} \quad (24)$$

This model involves a set of four simultaneous equations with partial derivatives in both time and distance. In order to complete the model, it is necessary to specify initial conditions (i.e., temperature profiles at time zero), and boundary conditions (i.e., two independent conditions on each of U, V, W, and Z for all time) must be provided.

Specification of the initial conditions is straightforward. The following boundary conditions on both pipe wall temperatures at the fluid inlet points are valid:

$$\partial W/\partial x = 0 \quad \text{at } x = 0 \quad (25)$$

$$\partial Z/\partial x = 0 \quad \text{at } x = L \quad (26)$$

The inlet fluid temperatures are prescribed functions of time. That is,

$$U = U(t) \quad \text{at } x = 0 \quad (27)$$

$$V = V(t) \quad \text{at } x = L \quad (28)$$

The four remaining boundary conditions involve fictitious extensions of the outlet ends of both the shell and tube. These kinds of boundary conditions in dispersion type models are discussed at length by Parulekar and Ramkrishna (1984). At some distance suitably far removed from the actual outlet points, a zero temperature gradient condition must apply. In these extensions, the fluids are assumed to be in contact with perfectly insulated walls. These conditions can be stated formally as follows:

$$\partial U/\partial x = 0 \quad \text{at } x = (n + 1)L \quad (29)$$

$$\partial V/\partial x = 0 \quad \text{at } x = -mL \quad (30)$$

$$\partial W/\partial x = 0 \quad \text{at } x = (n + 1)L \quad (31)$$

$$\partial Z/\partial x = 0 \quad \text{at } x = -mL \quad (32)$$

where n and m are suitably large positive numbers.

The model was solved using finite-difference analogs proposed by Price et al. (1966). A method was developed whereby only a few finite-difference grids were required in the fictitious extensions. The number of fictitious extensions required was determined by finding the minimum number that gave reproducible results within the actual exchanger. Typically only one or two extensions were required.

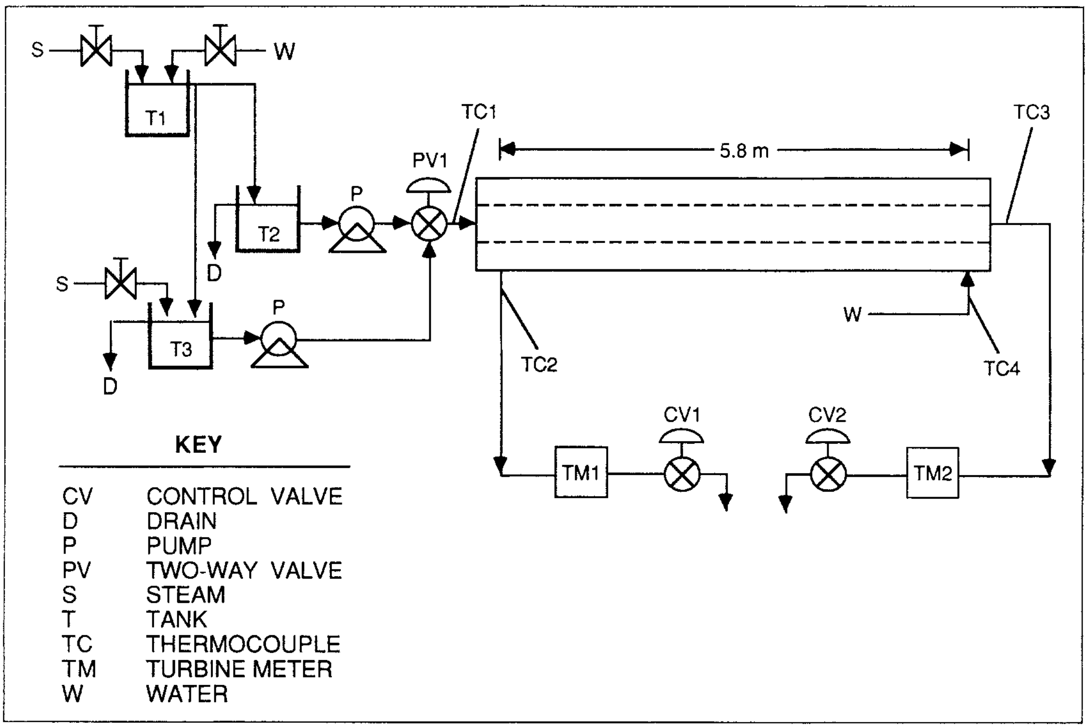

Figure 1. Heat exchanger system.

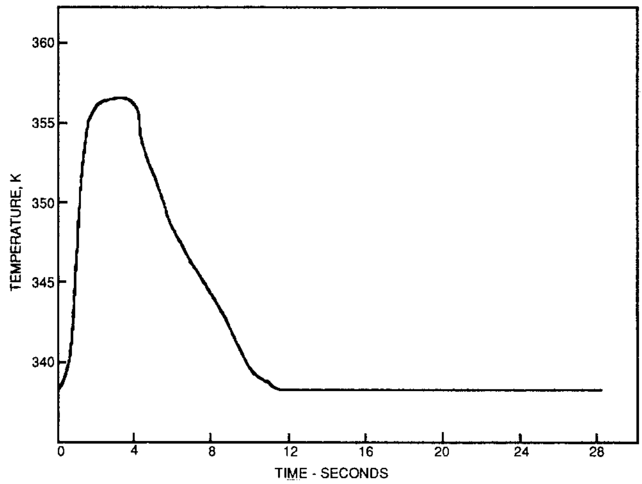

Figure 2. Tube temperature input pulse.

Comparison of Calculated and Experimental Results

This section presents a comparison between experimental data and results obtained from the solution of the mathematical model for the double-pipe heat exchanger as presented above. Of particular significance is the use of heat transfer coefficients calculated from well-known steady state correlations (Sieder and Tate, 1936), and apparent axial dispersion coefficients (for both the tube and shell sides of the exchanger) calculated from Eq. 17.

Beckman (1969) obtained dynamic response data on the shell and tube heat exchanger depicted schematically in Figure 1. The data for one set of experimental conditions given below are typical of several experimental runs made by Beckman.

A 20 K tube temperature pulse, plotted in Figure 2, was the transient disturbance from the following steady state conditions:

$$\text{Tube velocity} = 0.582\ \text{m} \cdot \text{s}^{-1}$$
$$\text{Shell velocity} = 1.22\ \text{m} \cdot \text{s}^{-1}$$
$$\text{Tube fluid inlet temp.} = 336.8\ \text{K}$$
$$\text{Tube fluid outlet temp.} = 314.5\ \text{K}$$
$$\text{Shell fluid inlet temp.} = 287.3\ \text{K}$$
$$\text{Shell fluid outlet temp.} = 298.0\ \text{K}$$

Water was used as the fluid in both the shell and tube sides.

The exchanger was constructed with a 2.54 cm nominal type B copper tube (ASTM B302) concentrically mounted inside a one and 1.26 cm nominal type B tube shell. The test section was 5.8 m long. The shell was insulated with 1.9 cm fiberglass pipe insulation. For further details of the experimental apparatus and procedures, see Beckman's original work (1969).

The experimental shell and tube outlet response data to the tube temperature forcing shown in Figure 2 are plotted in Figure 3 as discrete points.

Under the experimental conditions cited above, the system parameters were calculated; they are shown in Table 1. The dispersion coefficients were calculated from Eq. 17. Using these parameters in the mathematical mode of the exchanger, a numerical solution of the model was performed. The results of the numerical solution appear in Figure 3 as solid lines. For a

Table 1. System Parameters Corresponding to Experimental Conditions

	Tube Side	Shell Side
Re	32,760	13,216
Pr	3.407	7.316
T_b	324.8	293.2
D	56.82	46.37
h	0.8137	2.718

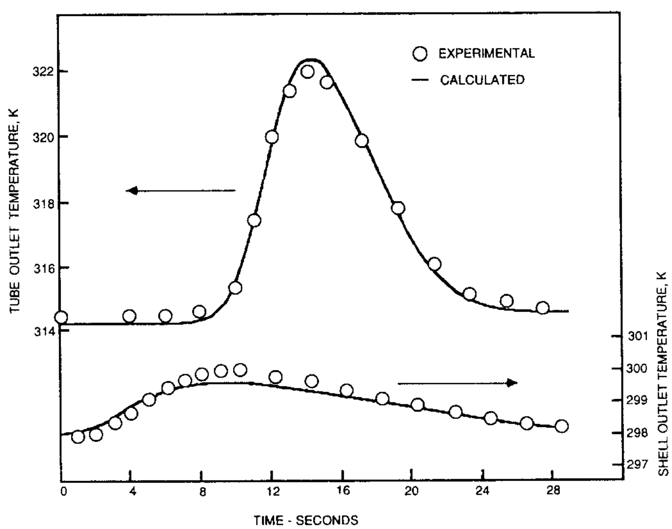

Figure 3. Results using dispersion coefficients from Eq. 17.

Table 2. Taylor Dispersion Coefficients

	Tube Side	Shell Side
D	5.495	5.000

description of the numerical techniques that were employed to obtain the solution to the partial differential equations of the model, see Beckman (1969) and Beckman and Law (1971).

The shell-side heat transfer coefficient was calculated using an equivalent diameter for heat transfer as suggested by Kern (1950). The shell-side dispersion coefficient was calculated using an equivalent diameter based on the total wetted perimeter.

Figure 3 shows excellent agreement between the experimental results and those predicted by the mathematical model using D and h values given in Table 1. The largest deviation on the tube side is 0.28 K and on the shell side is 0.33 K. These results are typical of several experimental cases considered by Beckman (1969).

In order to compare results using dispersion coefficients given by Eq. 17 and those predicted for adiabatic systems by Taylor, the model was solved again using Taylor's coefficients, whose values are shown in Table 2, with all other system parameters unchanged and as given in Table 1. These results appear in Figure 4 as solid lines.

Close examination of Figures 3 and 4 demonstrates clearly the superior agreement obtained using dispersion coefficients calculated from Eq. 17. It is significant that the better agreement shown in Figure 3 over that of Figure 4 is not simply in the magnitude of the peak temperature difference (which showed an improvement of nearly 2 K on the tube side) but also in the spread of the curves and even in the steady state values. The excellent agreement apparent in Figure 3 is typical of other comparisons made between simulation and experimental results. That is, the observed improvement was highly reproducible.

Conclusions

An expression has been derived for predicting apparent axial dispersion coefficients for use in one-dimensional models of systems in which transient disturbances have low energy content and, consequently, the transients do not significantly alter the radial temperature gradients from those caused by the steady state heat flux at the wall. This expression has been used in a particular application involving a double-pipe heat exchanger subjected to a tube inlet temperature forcing. Experimental data on the outlet temperatures of the exchanger agree quite well with those predicted by a mathematical model of the exchanger, which contained axial dispersion coefficients as predicted by the expression derived in this paper and heat transfer coefficients predicted by well-known steady state correlations. For the purposes of simulating the dynamics of the double-pipe heat exchanger for practical applications, these results are considered excellent. The significance of the good agreement between experimental and calculated responses is that Eq. 17 gives a convenient and reliable means of predicting apparent axial dispersion coefficients in the presence of large radial gradients. It is expected that this result would be equally applicable in geometries other than simple double-pipe heat exchangers.

Acknowledgment

The support of the National Science Foundation, Grant No. GK-1329, is gratefully acknowledged.

Notation

a = pipe radius, m
A = cross-sectional area, m^2
c_p = heat capacity at constant pressure, J · g^{-1}
D = axial dispersion coefficient, W · m^{-1} · K^{-1}
D_L = apparent axial dispersion coefficient with large radial gradient, W · m^{-1} · K^{-1}
DT = adiabatic axial dispersion coefficient (Taylor, 1954), W · m^{-1} · K^{-1}
$f(z)$ = logarithmic function from von Karman's generalized velocity profiles
$f'(z)$ = $df(z)/dz$

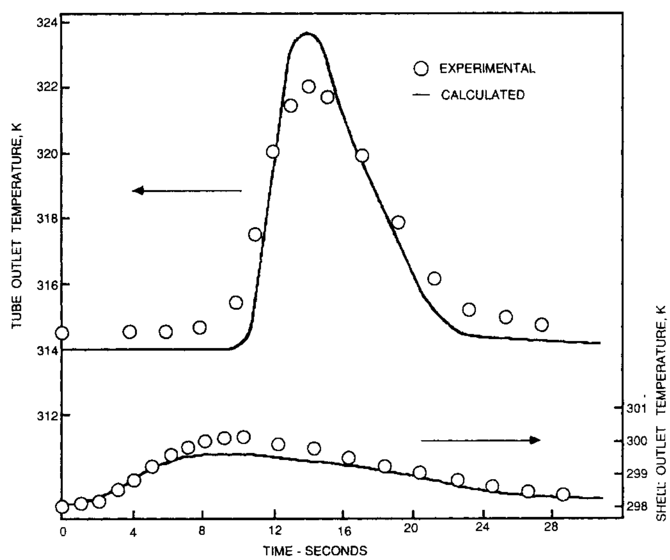

Figure 4. Results using Taylor dispersion coefficients.

F = Fanning friction factor
h = heat transfer coefficient, $W \cdot m^{-2} \cdot K^{-1}$
k = thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
L = length of exchanger test section, m
m = number of fictitious extensions on shell side
n = number of fictitious extensions on tube side
P = perimeter, m
Pr = Prandtl number
q = radial heat flux from tube wall to fluid in the tube, $J \cdot s^{-1} \cdot m^{-2}$
Q = longitudinal rate of heat transfer by dispersion, $J \cdot s^{-1}$
r = radial coordinate, m
Re = Reynolds number
t = time, s
T = temperature (time-smoothed), K
T_0 = T at $z = 0$
T_b = averaged bulk fluid temperature, K
u = velocity (time-smoothed), $m \cdot s^{-1}$
U = tube fluid temperature (time-smoothed), K
v = mean fluid velocity, $m \cdot s^{-1} = \int_0^1 uz\,dz \Big/ \int_0^1 z\,dz$
v_* = friction velocity $= v\sqrt{F/2}$
V = shell fluid temperature (time-smoothed), K
W = tube wall temperature (time-smoothed), K
x = stationary axial coordinate, m
z = r/a
Z = shell wall temperature (time-smoothed), K

Greek letters

α = molecular diffusivity of heat
ϵ = turbulent transport coefficient
ρ = density

Subscripts

1 = tube fluid
2 = tube wall
3 = shell fluid
4 = shell wall

Literature Cited

Aris, R., "On the Dispersion of a Solute in a Fluid Flowing through a Tube," *Proc. Roy. Soc.*, **A235,** 67 (1956).

Beckman, L. V., "Unsteady State Simulation and Control Study of a Counterflow Distributed Parameter System: The Double-Pipe Heat Exchanger," Ph.D. Diss., Tulane Univ., New Orleans (1969).

Beckman, L. V., and V. J. Law, "Unsteady State Simulation of the Counterflow Double-Pipe Heat Exchanger," *Proc. IFAC Symp.*, Banff, Canada, Paper 6-3 (June, 1971).

Gill, W. N., "A Note on the Solution of Transient Dispersion Problems," *Proc. Roy. Soc.*, **A298,** 335 (1967).

Gill, W. N., and R. Sankarasubramanian, "Exact Analysis of Unsteady Convective Diffusion," *Proc. Roy. Soc.*, **A316,** 341 (1970).

———, "Dispersion of a Non-Uniform Slug in Time-Dependent Flow," *Proc. Roy. Soc.*, **A322,** 101 (1971).

Goldschmidt, V. W., and M. K. Householder, "Longitudinal Dispersion for Turbulent Flow in Pipes," *Ind. Eng. Chem. Fundam.*, **8,** 172 (1969).

Kern, D. G., *Process Heat Transfer*, McGraw-Hill, New York, 105 (1950).

Parulekar, S. J., and D. Ramkrishna, "Analysis of Axially Dispersed Systems with General Boundary Conditions," *Chem. Eng. Sci.*, **39,** 1571 (1984).

Price, H. S., R. S. Varga, and J. E. Warren, "Application of Oscillation Matrices to Diffusion-Convection Equations," *J. Math Physics*, **45,** 301 (1966).

Rodehorst, C. W., and V. J. Law, "A One-Dimensional Dispersion Model for Transient Heat Transfer in Turbulent Pipe Flow," *Proc. 5th Int. Heat Transfer Conf.*, Paris (1970).

Sieder, E. N., and G. E Tate, "Heat Transfer and Pressure Drop of Liquids in Tubes," *Ind. Eng. Chem.*, **28,** 1429 (1936).

Sittel, C. N., Jr., W. D. Threadgill, and K. B. Schnelle, Jr., "Longitudinal Dispersion for Turbulent Flow in Pipes," *Ind. Eng. Chem. Fundam.*, **7,** 39 (1968).

Taylor, G. I., "The Dispersion of Matter in Turbulent Flow Through Pipes," *Proc. Roy. Soc.*, **A223,** 446 (1954).

Vrentas, J. S., and C. M. Vrentas, "Dispersion in Laminar Tube Flow at Low Peclet Numbers or Short Times," *AIChE J*, **34,** 1423 (1988).

Manuscript received Mar. 3, 1989, and revision received Feb. 5, 1990.